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IBM-2 CALCULATION OF $^{108-116}\text{Cd}$ NUCLEI

Key words: The interacting boson model-2, IBM, NPBOS code, excitation energies, electromagnetic transition strength, static moments

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ABSTRACT

Excitation energies, electromagnetic transition strength, and static moments in $^{108,110,112,114,116}\text{Cd}$ nuclei are studied in the proton-neutron interacting boson model-2 (IBM-2). The calculations were carried out by using an improved version of the NPBOS code. The calculated values for Cd isotopes and the experimental ones agree. For the excitation energies the splitting of the 2-phonon multiplet was found rather small. In order to locate the Cd nuclei on the map of the IBM-2 parameters, the branching ratios were examined. It is found that Cd nuclei are classified in the U(5) limit.

INTRODUCTION

In the early days of the interacting boson model (IBM-1) [1, 2] achieved much success by providing a unified, phenomenological description of low-lying, collective states exhibited by many medium-to-heavy mass nuclei. The impressive phenomenological success of the IBM-1 provoked a search for an underlying basis for the model. From a microscopic viewpoint there is strong justification for invoking separate neutron and

proton degrees of freedom, which is not done in the IBM-1. Furthermore, the IBM-2 [2, 3] was developed to have distinct neutron bosons and proton bosons. The result is a model, linked to the underlying shell model, that reproduces a wide range of collective features and is capable of distinguishing neutron and proton characteristics.

IBM-2 has two distinctive advantages over the more conventional description of collective states. First of all, it describes the proton and neutron degrees of freedom separately. Secondly, a clear relationship with microscopic theories have been established [4].

This relationship enables one to drive in a simple way values of the parameters appearing in the IBM-2 hamiltonian, when degenerate single-particle levels are assumed for the valence nucleons. Differences between the fitted values of these parameters and degenerate values can then be interpreted in terms of the actual non-degeneracy of the single-particle levels.

Cadmium isotopes have been the subject of studies in nuclear-structure physics [5, 6, 7]. Cadmium has 48 protons and is close to the $Z=50$ shell closure. Hence, it has been considered as an anharmonic vibration-like nucleus. This corresponds to the (U5) limit in the interacting boson model (IBM).

The aims of the present work are the following:

- (i) To carry out systematic IBM-2 calculations of $^{108-116}\text{Cd}$, the attention being paid to excitation energies, electromagnetic transition rates, and quadrupole moments.
- (ii) To locate the Cd nuclei on the map of the IBM-2 parameters.
- (iii) To describe the main features of Cd isotopes with $108 \leq A \leq 116$, choosing the right parameters in all calculations.

Numerical calculations have been carried out by using an improved version of the NPBOS code [8].

THE MODEL

The microscopic picture of the IBM is given in terms of collective pairs of nucleons [9]. One assumes that the valence protons or neutrons outside a major closed shell can be treated as proton or neutron bosons, by considering them in pairs. In this way, the nucleus $^{108}_{48}\text{Cd}_{60}$, for example, is described by 1 proton boson hole and 5 neutron boson particles.

Table-1. NPBOS Parameters (All parameters in MeV , with the exception of χ_v, χ_π)

	ε	κ	χ_v	χ_π	ξ_1	ξ_2	ξ_3	C0N	C2N	C4N
^{108}Cd	0.840	-0.190	-0.500	-1.000	0.000	0.080	0.000	0.100	0.050	0.050
^{110}Cd	0.960	-0.190	-0.200	-0.950	0.010	0.010	0.010	0.000	0.000	0.000
^{112}Cd	0.880	-0.150	-0.100	-1.000	0.250	0.020	0.250	-0.350	-0.150	0.000
^{114}Cd	0.820	-0.150	-0.150	-0.900	0.200	0.080	0.080	-0.300	-0.100	0.000
^{116}Cd	0.780	-0.170	-0.100	-0.900	0.000	0.000	0.100	-0.100	0.000	0.020

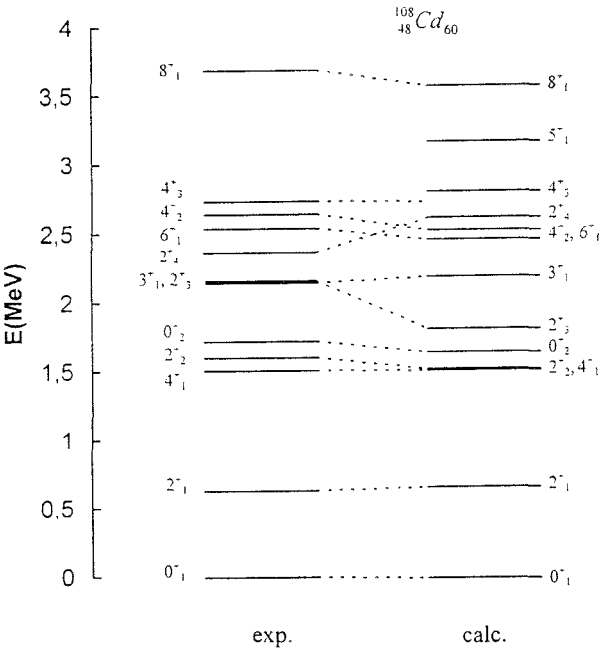


Figure-1. Experimental and calculated energy levels in $^{108}_{48}\text{Cd}_{60}$. Experimental data are taken from ref. [11].

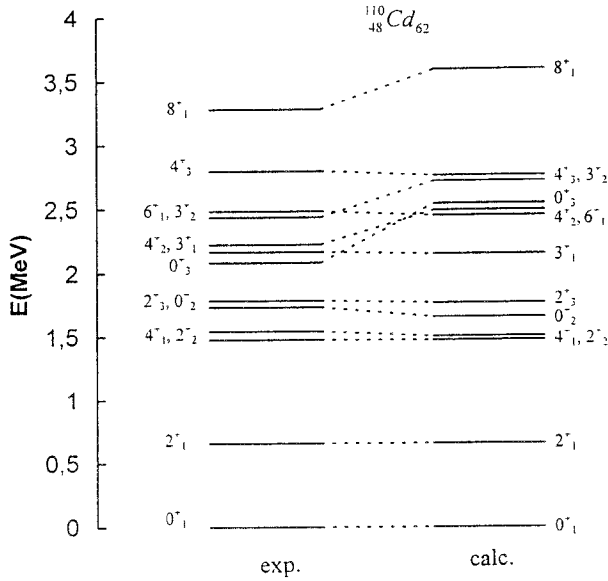


Figure-2. Experimental and calculated energy levels in $^{110}_{48}\text{Cd}_{62}$. Experimental data are taken from ref. [12].

Only pairs coupled to $L = 0$ and $L = 2$ are considered. These pairs correspond intuitively to the s and d bosons, respectively.

The neutron-proton interacting boson model (IBM-2) hamiltonian we use has the following expression [10]:

$$H = \varepsilon_{\pi} d_{\pi}^{\dagger} d_{\pi} + \varepsilon_{\nu} d_{\nu}^{\dagger} \tilde{d}_{\nu} + V_{\pi\pi} + V_{\nu\nu} + \kappa Q_{\pi} \cdot Q_{\nu} + M_{\pi\nu} \quad (1)$$

where

$$Q_{\rho} = (s_{\rho}^{\dagger} \tilde{d}_{\rho} + d_{\rho}^{\dagger} s_{\rho})^{(2)} + \chi_{\rho} (d_{\rho}^{\dagger} \tilde{d}_{\rho})^{(2)}, \quad \rho = \pi, \nu \quad (2)$$

$$V_{\rho\rho} = \sum_{l=0,2,4} \frac{1}{2} C_{l_{\rho}} (2L+1)^{1/2} [(d_{\rho}^{\dagger} d_{\rho})^{(2)} (\tilde{d}_{\rho} \tilde{d}_{\rho})^{(2)}]^{(0)}, \quad \rho = \pi, \nu \quad (3)$$

$$M_{\pi\nu} = - \sum_{\kappa=1,3} 2\xi_{\kappa} (d_{\pi}^{\dagger} d_{\nu})^{(\kappa)} \cdot (\tilde{d}_{\pi} \tilde{d}_{\nu})^{(\kappa)} + \xi_2 (d_{\pi}^{\dagger} s_{\nu}^{\dagger} - s_{\pi}^{\dagger} d_{\nu}^{\dagger}) \cdot (\tilde{d}_{\pi} s_{\nu} - s_{\nu} \tilde{d}_{\nu})^{(2)}. \quad (4)$$

Here ε_{π} and ε_{ν} stand for the energies of the proton and neutron bosons, respectively. $V_{\pi\pi}$ and $V_{\nu\nu}$ indicate the interactions between similar bosons and the term $\kappa Q_{\pi} \cdot Q_{\nu}$ denotes the

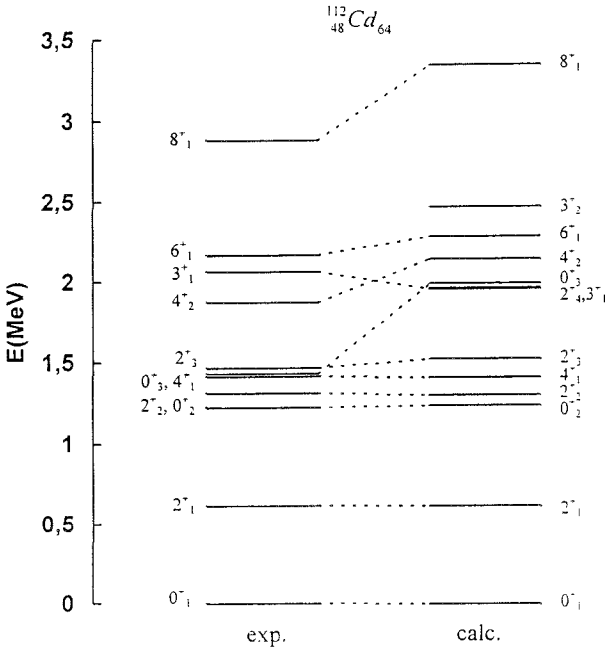


Figure-3. Experimental and calculated energy levels in $^{112}_{48}\text{Cd}_{64}$. Experimental data are taken from ref. [13].

quadrupole-quadrupole interaction between proton and neutron bosons. $M_{\pi\nu}$ is an operator which affects the energy of those states which are not fully symmetric in the neutron-proton degree of freedom.

The $B(E2)$'s were calculated by using the operator

$$T(E2) = e_\pi Q_\pi + e_\nu Q_\nu \tag{5}$$

where e_π and e_ν are boson effective charges.

The $M1$ operator is given by

$$T(M1) = \sqrt{\frac{3}{4\pi}} (g_\pi \hat{L}_\pi + g_\nu \hat{L}_\nu) \tag{6}$$

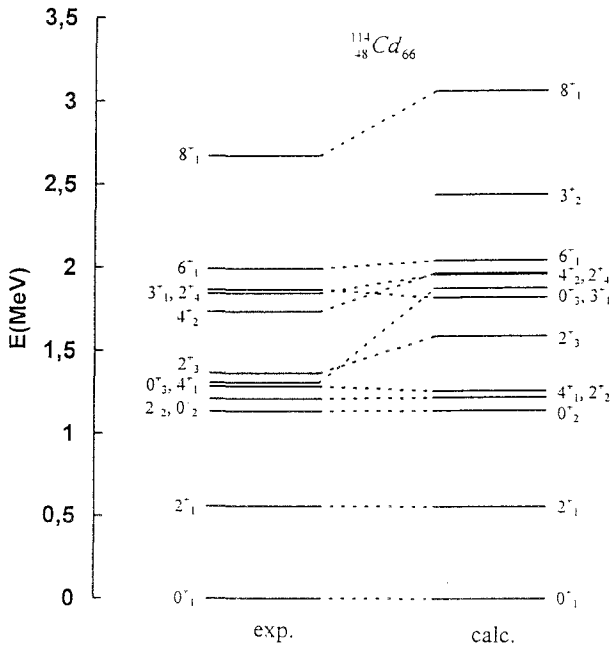


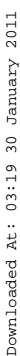
Figure-4. Experimental and calculated energy levels in $^{114}_{48}\text{Cd}_{66}$. Experimental data are taken from ref. [14].

where $\hat{L}_\rho = \sqrt{10}(\tilde{d}_\rho \times \tilde{d}_\rho)^{(1)}$ ($\rho=\pi,\nu$) is the angular momentum operator of either kind of bosons, and g_π and g_ν are the proton and neutron boson g -factors, respectively.

RESULTS

We used parameters for the IBM-2 calculations of Cd isotopes which are given in Table-1. The parameters in the hamiltonian were gradually varied to produce the best fit to experimentally determined energy levels, $B(E2)$'s, $B(M1)$'s, and quadrupole moments simultaneously.

The calculated excitation energies for even $^{108-116}\text{Cd}$ as well as the experimental ones are shown in Figs. 1, 2, 3, 4, and 5, respectively. The general agreement between



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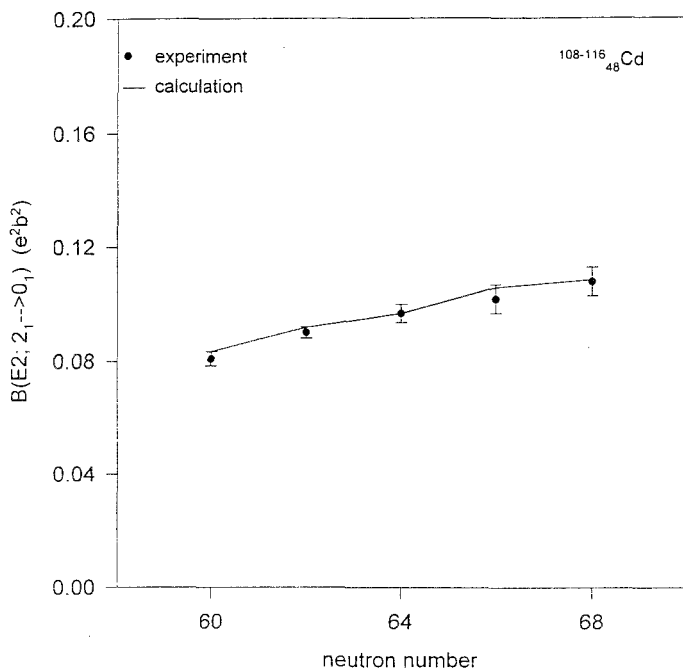


Figure-6. Relation between $B(E2; 2_1^- \rightarrow 0_1^+)$ and neutron number for the $^{108-116}_{48}\text{Cd}$ nuclei. Experimental data are taken from references [11, 12, 13, 14, 15].

experiment and calculation is very good. The calculated yrast 8^+ excitation energies are slightly higher than the experimental ones, which is a general feature of this type of model. The splitting of the two-phonon multiplet is rather small.

For calculating E2 transition rates and the quadrupole moment of the first excited 2^+ state, we used the boson E2 operator, which is given in Equa. 5. In principle, the values e_π and e_v could be different from each other and different for each nucleus. Microscopic calculations indicate that $e_\pi \approx 2e_v$ in spherical nuclei. The value of e_π and e_v , which are given in Table- 2, were determined from the experimental $B(E2; 2_1^+ \rightarrow 0_1^+)$ in Cd nuclei. In Figs. 6 and 7 we shown the $B(E2; 2_1^+ \rightarrow 0_1^+)$ and $B(E2; 4_1^+ \rightarrow 2_1^+)$ values, which are of the

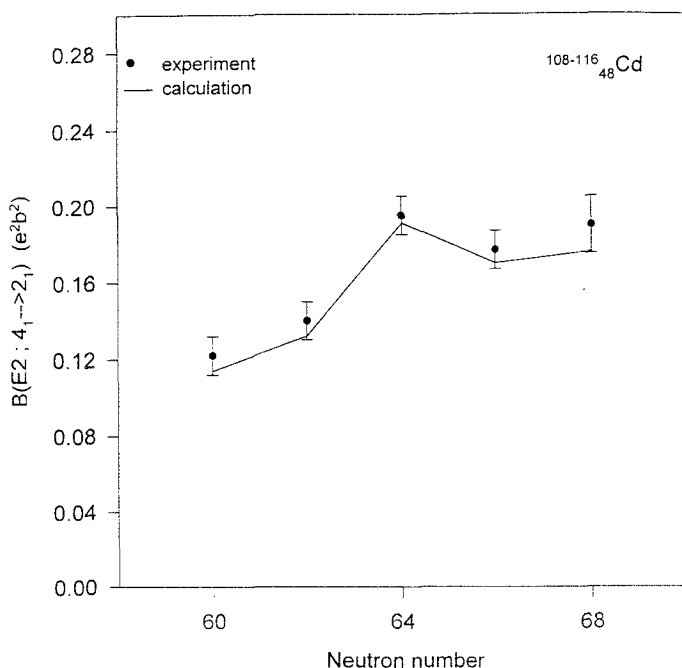


Figure-7. Relation between $B(E2; 4_1^- \rightarrow 2_1^-)$ and neutron number for the $^{108-116}_{48}\text{Cd}$ nuclei. Experimental data are taken from references [11, 12, 13, 14, 15].

same order magnitude and display a typical increase towards the middle of the shell. In Fig. 8 we show the $B(E2; 2_2^+ \rightarrow 2_1^+)$ values as a function of neutron number. Because of possible M1 admixture, this quantity is rather difficult to measure. $B(E2)$ values are given in Table-3.

To conclude on the E2 properties, we give the results for the quadrupole moments $Q_{2_1^+}$ of the first excited 2^+ state in Cd isotopes as a function of neutron number as in Table-4. The calculated $Q_{2_1^+}$'s are found consistent with the experimental ones both in sign and in magnitude. The $Q_{2_1^+}$ values are given in Fig.9.

The $B(M1; 2_2 \rightarrow 2_1)$ transition rates were well reproduced by the calculations. Figure 10 shows this agreement. The experimental and the calculated $B(M1; 2_2 \rightarrow 2_1)$ transition rates are given in Table- 5.

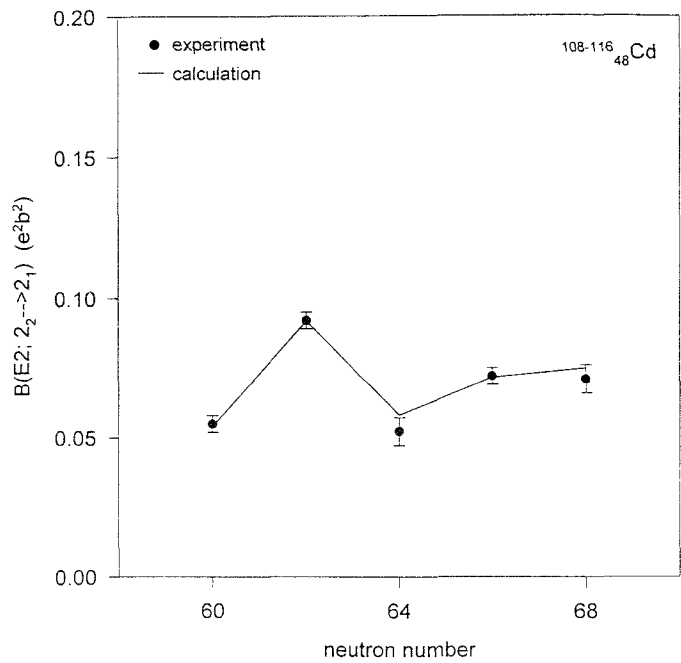


Figure-8. Relation between $B(E2; 2_2 \rightarrow 2_1)$ and neutron number for the $^{108-116}_{48}\text{Cd}$ nuclei. Experimental data are taken from references [11, 12, 13, 14, 15].

Table-3. $B(E2)$ values (in e^2b^2) of $^{108-116}\text{Cd}$ nuclei.

Isotopes	$2_1 \rightarrow 0_1$		$4_1 \rightarrow 2_1$		$2_2 \rightarrow 2_1$	
	exp.	cal.	exp.	cal.	exp.	cal.
^{108}Cd	0.081 ± 0.01	0.083	0.122 ± 0.01	0.114	0.055 ± 0.01	0.054
^{110}Cd	0.090 ± 0.01	0.092	0.144 ± 0.01	0.132	0.092 ± 0.01	0.092
^{112}Cd	0.097 ± 0.01	0.097	0.195 ± 0.01	0.191	0.051 ± 0.01	0.058
^{114}Cd	0.101 ± 0.01	0.106	0.177 ± 0.01	0.170	0.072 ± 0.01	0.072
^{116}Cd	0.108 ± 0.01	0.108	0.195 ± 0.01	0.176	0.071 ± 0.01	0.076

Table-4. The $Q_{2_1^+}$ values (in eb)

Isotopes	Q_{exp}	$Q_{cal.}$
¹⁰⁸ Cd	-0.45	-0.44
¹¹⁰ Cd	-0.40	-0.40
¹¹² Cd	-0.37	-0.38
¹¹⁴ Cd	-0.36	-0.35
¹¹⁶ Cd	-0.42	-0.42

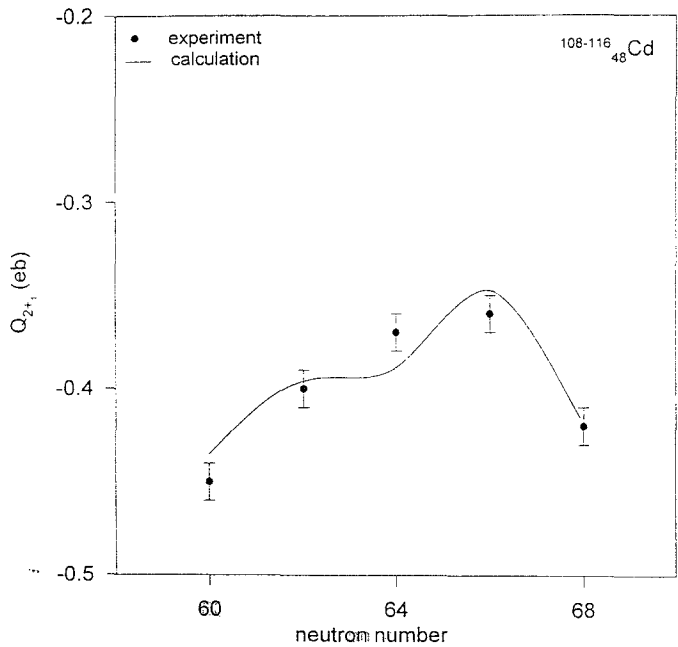


Figure-9. Comparison between experimental and calculated quadrupole moments of the 2_1^+ for the $^{108-116}\text{Cd}$ nuclei. Experimental data are taken from references [11, 12, 13, 14,15].

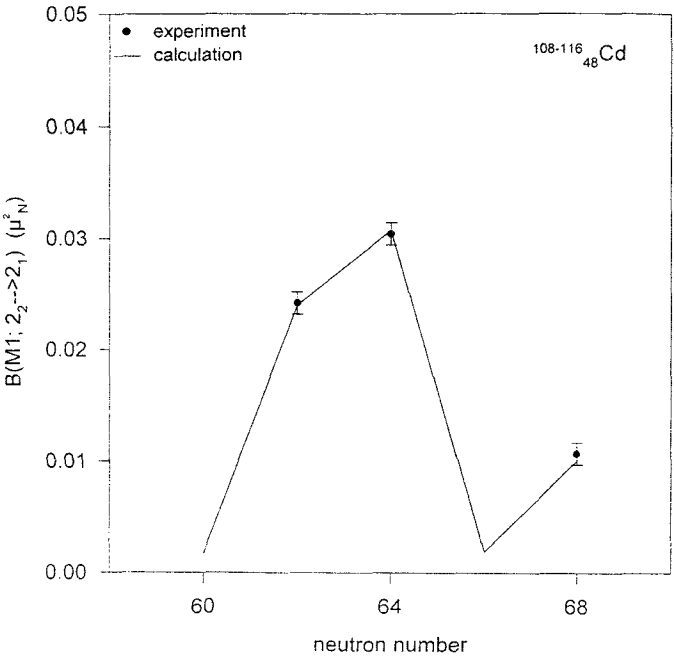


Figure-10. Relation between $B(M1;2_2\rightarrow2_1)$ and neutron number for the Cd nuclei. Experimental data are taken from references [12, 13, 15].

Table 5. $B(M1;2_2 \rightarrow 2_1)$ values (in μ_N^2)

Isotopes	$B(M1;2_2 \rightarrow 2_1)$	
	exp.	cal.
^{108}Cd	≤ 0.033	0.0017
^{110}Cd	0.0242 ± 0.005	0.0240
^{112}Cd	0.0304 ± 0.005	0.0307
^{114}Cd	-	0.0019
^{116}Cd	0.0107 ± 0.005	0.0101

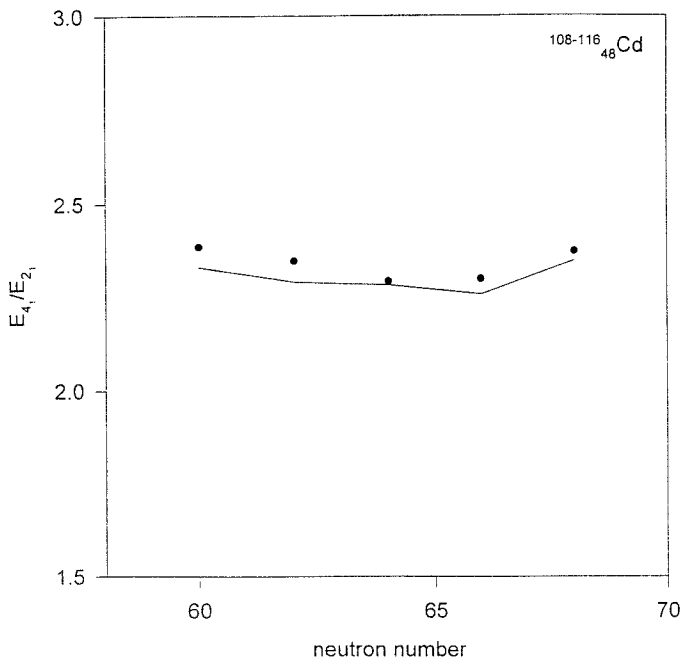


Figure-11 The E_{4_1}/E_{2_1} energy ratio for the $^{108-116}_{48}\text{Cd}$ nuclei. The continuous line represents calculation, the dots represent experiment. Experimental data are taken from references [11, 12, 13, 14,15].

The energy ratios $E(4_1)/E(2_1)$ indicate the U(5) limit for $^{108-116}\text{Cd}$ nuclei. The $E(4_1)/E(2_1)$ ratio is plotted in Fig. 11. The agreement between the experimental values and the calculated ones is very good. The variation of both the experimental and the calculated ratios is rather small.

CONCLUSIONS

We have presented here the results of a study of some Cd isotopes in the framework of the interacting boson model-2. The results of this work show that the IBM-2 provides a good description of $^{108-116}\text{Cd}$ nuclei, provided the parameters used in the calculation are chosen in the right way.

The calculated excitation energies for Cd nuclei as well as the experimental ones are in agreement. The energy levels of one and two-phonon states are calculated in good agreement with the experimental data. The calculated yrast 8_1^+ excitation energies are slightly higher than the experimental ones, which is a general feature of the interacting boson approximation. It is shown that the energy levels drop in the middle of the shell. Since $Z=48$ is close to the $Z=50$ shell, the protons may jump across the $Z=50$ shell. Therefore, the 2p-2h states (or core excitation) are important in the low-lying levels.

The B(E2) and B(M1) transition rates are well reproduced by the calculation. It is found that $^{108-116}\text{Cd}$ nuclei are classified in the U(5) limit.

Q_{2f} is not a monotonic function of the mass number. As a matter of fact, Q depends sensitively on κ and $(\chi_v + \chi_\pi)$ in Cd nuclei.

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